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Math 118 - Spring 2023 - Common Final Exam, version A

Print name:____

Section number:__________ **Instructor's name:**____________________________

Directions:

- This exam has 13 questions. Please check that your exam is complete, but otherwise keep this page closed until the start of the exam is called.
- Fill in your name, and your instructor's name.
- It will be graded out of 100 points.
- Show your work. Answers (even correct ones) without the corresponding work will receive no credit.
- A formula sheet has been provided with this exam. You may not refer to any other notes during the exam.
- You may use a calculator which does not allow internet access. The use of any notes or electronic devices other than a calculator is prohibited.
- **Unless otherwise stated, round any constants to two decimal places if necessary.**

Good luck!

Question:	1	2	3	4	5	6	7
Points:	9	9	7	3	6	8	8
Score:							

Question:	8	9	10	11	12	13	Total
Points:	12	6	6	9	8	9	100
Score:							

1. (9 points) The output of Adnan's potatoes farm is 1500 potatoes in the year 2023. Recall that a linear function has a general form of $P = mt + b$ and an exponential function has a general form of $P = a \cdot b^t$.

   (a) If Adnan increases his potato output by a rate of 100 potatoes per year, find a formula for the function $P(t)$, the number of potatoes t years after 2023.

   (b) If the output is increasing by 16% per year, find a formula for the function $P(t)$, the number of potatoes t years after 2023.

   (c) Under the assumptions stated in part b, find the year that potato output will hit 5000. Round to the nearest whole number.

2. (9 points) Dane opens a bank account with an initial deposit of \$7000. It earns interest at a nominal rate of 5% per year. Find the balance of their account after 6 years if interest is compounded as follows.

   (a) Annually (once a year).

   (b) Quarterly (four times per year).

   (c) Continuously.

3. (7 points) Consider the exponential function $Q = 4.2(0.182)^t$.

   (a) Give the initial value, growth factor, and growth rate for the given function.

   The initial value is ________

   The growth factor is ________

   The growth rate is ________

   (b) Write the given function in the form $Q = ae^{kt}$.

   (c) Determine if this function displays exponential growth or decay. Explain your answer in a sentence.
   Circle one: **exponential growth** or **exponential decay**.

4. (3 points) Give the domain and range for the function $f(x) = 12e^{-2.3x}$.

   The domain is ________

   The range is ________

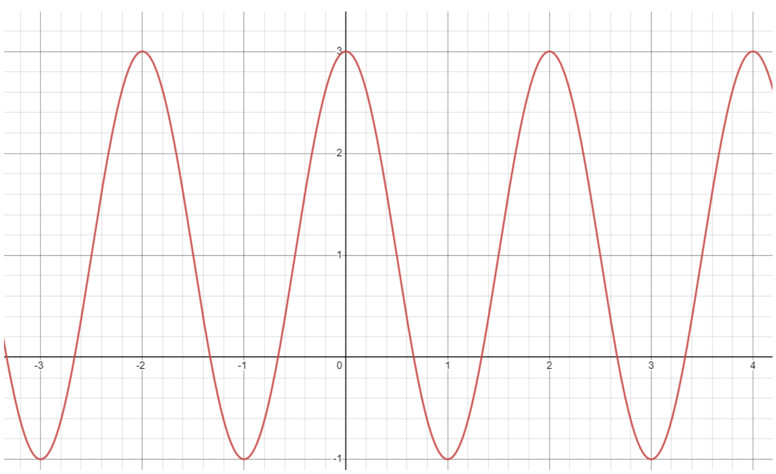
5. (6 points) The chemistry department discovers a new element called "Lucnium" and wants to find out more about it. Lucnium decays at a continuous rate of 8% per hour. Find the half-life of Lucnium. Make sure to include units in your answer.

6. (8 points) On August 22, 2022 high tide in Chicago was at midnight. The water level at high tide was 9.6 feet; later, at low tide, it was 0.4 feet. Assume the next high tide is at exactly 1 PM.

   (a) Use a sinusoidal function to model the height of the water level in Chicago on August 22nd, 2022 as a function of time, t, the number of hours since midnight.

   (b) Write an equation for the first time the tide is 7 feet high after midnight. Find a solution to this equation, giving your answer in terms of an inverse trig function and evaluate with correct units.

7. (8 points) Find a formula of the trigonometric function shown in the graph below.



8. (12 points) For angles α and β such that $0 < \alpha < \frac{\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$ such that $\sin(\alpha) = \frac{3}{5}$ and $\cos(\beta) = -\frac{2}{7}$. Find the given quantities without finding α and β. Give an exact answer.

(a) $\cos(\alpha)$

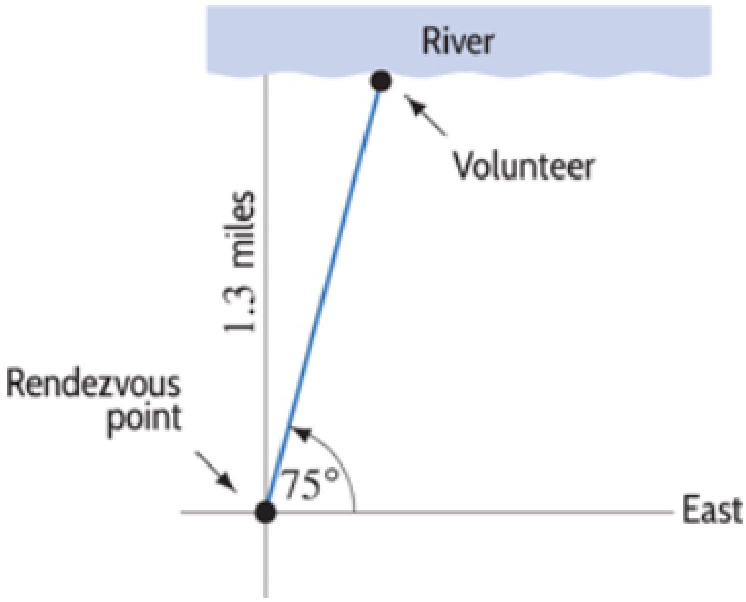
(b) $\sin(\beta)$

(c) $\sin(\alpha + \beta)$

(d) $\cos(\alpha + \beta)$

9. (6 points) Determine if the identity $(\sin^2(\theta) + \cos^2(\theta))(\tan(\theta)\cot(\theta)) + 4\cos(\theta)\sec(\theta) = 5$ is true (that is, for any value of θ). Show work to support your answer.

10. (6 points) A search and rescue volunteer leaves a rendezvous point in the Arizona desert walking 75 degrees north of east. She reaches a river that runs east-west and is located 1.3 miles directly north of the rendezvous point. See the figure below. How far east of the rendezvous point is the volunteer when she reaches the river? Make sure to include units in your answer.



11. (9 points) Let $P = f(t) = 600(1.328)^t$ be the population of people that live in a town, where t is measured in years since 2020.

    (a) Evaluate $f(6)$. Round to the nearest whole number. Describe in words what this quantity represents. Write your answer in a complete sentence with units.

    (b) Find a formula for $f^{-1}(P)$ in terms of P. Give an exact answer.

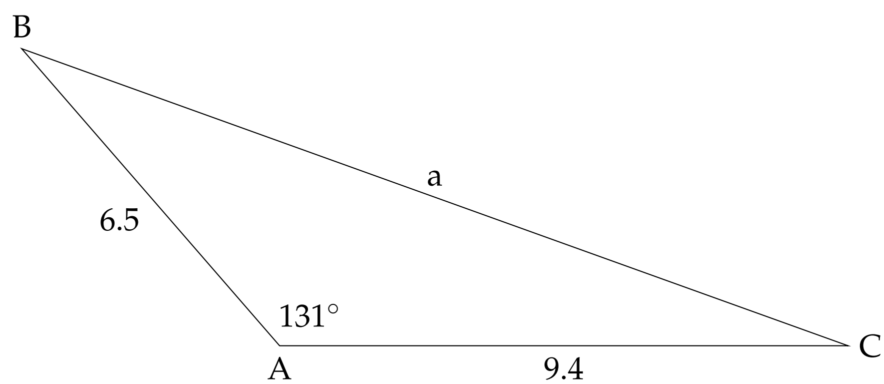
    (c) Evaluate $f^{-1}(2000)$. Round to the nearest whole number. Describe in words what this quantity represents. Write your answer in a complete sentence with units.

12. (8 points) Let $f(x) = 2x + 5$, $g(x) = 3x - 9$ and $h(x) = \ln(x)$. Find the following:

    (a) $f(g(4))$

    (b) $h(f(g(x)))$

13. (9 points) Find the length of the missing side, a, and the missing angle measures, B and C, in the diagram below. Give angles as degrees.



Exponential and Logarithm Formulas
Linear Function: $Q(t) = mt + b$
Exponential Function: $Q(t) = a \cdot b^t$
Continuous Exponential Function: $Q(t) = a \cdot e^{kt}$
Simple Interest: $B = P(1 + r)^t$
Compound Interest: $B = P\left(1 + \frac{r}{n}\right)^{nt}$

Trigonometry Formulas
1 radian $= \frac{180}{\pi}$ degrees and 1 degree $= \frac{\pi}{180}$ radians

$$\sin(\theta) = \frac{opp}{hyp} = \frac{y}{r} \quad \cos(\theta) = \frac{adj}{hyp} = \frac{x}{r} \quad \tan(\theta) = \frac{opp}{adj} = \frac{y}{x} = \frac{\sin(\theta)}{\cos(\theta)}$$

$$\csc(\theta) = \frac{1}{\sin(\theta)} \quad \sec(\theta) = \frac{1}{\cos(\theta)} \quad \cot(\theta) = \frac{1}{\tan(\theta)} = \frac{\cos(\theta)}{\sin(\theta)}$$

Pythagorean Identities: $\sin^2(\theta) + \cos^2(\theta) = 1 \quad \tan^2(\theta) + 1 = \sec^2(\theta) \quad 1 + \cot^2(\theta) = \csc^2(\theta)$

Sum and Difference Formulas:
$\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B)$
$\sin(A - B) = \sin(A)\cos(B) - \cos(A)\sin(B)$
$\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$
$\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$

Even-Odd Identities: $\sin(-x) = -\sin(x)$ and $\cos(-x) = \cos(x)$

Other identities: $\sin(\theta) = \sin(180^\circ - \theta)$, $\cos(\theta) = -\cos(180^\circ - \theta)$ and $\tan(\theta) = -\tan(180^\circ - \theta)$

General form for sine and cosine: $f(t) = A\sin(Bt) + k$ and $f(t) = A\cos(Bt) + k$

General form with horizontal shift: $f(t) = A\sin(B(t - h)) + k$ and $f(t) = A\cos(B(t - h)) + k$)

Period for sine and cosine: $P = \frac{2\pi}{|B|}$

Law of Sines: $\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$

Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos(C)$

Arc Length: $s = r\theta$

Inverse Trig Functions
$\theta = \cos^{-1}(y)$ provided that $y = \cos(\theta)$ and $0 \leq \theta \leq \pi$
$\theta = \sin^{-1}(y)$ provided that $y = \sin(\theta)$ and $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$
$\theta = \tan^{-1}(y)$ provided that $y = \tan(\theta)$ and $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

Polar coordinates conversions
$r^2 = x^2 + y^2$, $\tan(\theta) = \frac{y}{x}$, $x = r\cos(\theta)$, $y = r\sin(\theta)$

The Unit Circle